Chaos in East European black market exchange rates

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Summary

In this paper we test for deterministic chaos in seven East European black market exchange rates, using Koedijk and Kooi's (1992, Journal of Business and Economic Statistics, 10, 83–96) monthly data from January 1955 through May 1990. In doing so we use three (non-parametric) inference methods, the BDS (Brock et al., 1996, Econometric Reviews, 15, 197–235) test for whiteness, the Lyapunov exponent estimator of Nychka et al. (1992, Journal of the Royal Statistical Society, 12, 135–136) as well as the Lyapunov exponent estimator of Gencay and Dechert (1992, Physica D, 59, 142–157). We find some consistency in inference across methods, and we conclude, based on the Nychka et al. (1992) estimator, that there is evidence consistent with a chaotic non-linear generation process in only two out of seven series.


Keywords: Chaos, non-linearity, exchange rates.

1. Introduction

A general consensus has emerged in recent years that exchange rate series have a unit root in their univariate time-series representation, implying that exchange rate movements are better characterized as being the sum of permanent and transitory components where the permanent component is a random walk. Although the economic significance of this distinction is a subject of continuing debate, see, for example, Cochrane (1991) and Christiano and Eichenbaum (1989), there is also evidence that these random walk components are not different but perhaps arise from the response to the same set of fundamentals—see Baillie and Bollerslev (1989) and Serletis (1994).

While most of the empirical tests of the random walk model are based on linear models, interest in deterministic chaotic dynamics
has in the recent past experienced a tremendous rate of development and the literature is still growing. Chaos is a non-linear deterministic process which looks random. In fact, chaotic processes may have first and second moment properties that are the same as for white noise processes—this is why they are also called “white chaos”. The distinguishing feature of chaotic systems is, however, that they exhibit sensitive dependence on initial conditions, meaning that nearby identical chaotic systems in slightly different states will rapidly evolve toward very different states—see Medio (1992) for details.

For more than a century, chaotic dynamics has been studied almost exclusively by theoreticians. However, theorizing might be viewed (by economists) as empty if there is no evidence of chaos in macroeconomic and financial time series. Therefore, a number of researchers have recently focused on testing for non-linearity in general and chaos in particular in economic time series, with particularly encouraging results. For example, Barnett and Chen (1988) claimed successful detection of chaos in the (demand-side) Divisia monetary aggregates and their conclusion was further confirmed by DeCoster and Mitchell (1991, 1994), although it was disputed by Ramsey et al. (1990) and Ramsey and Rothman (1994). Further results relevant to this controversy have recently been provided by Serletis (1995). Moreover, the analysis of financial time series has led to results which are, as a whole, more interesting and more reliable than those of macroeconomic series—see, for example, Scheinkman and LeBaron (1989), Frank and Stengos (1989) and Hsieh (1991). Medio (1992, Chapter 14) provides an excellent survey of the literature concerning chaotic dynamics in macroeconomic and financial data.

The purpose of this paper is to test for determinist chaos in seven East European black market exchange rates, using Koedijk and Kool’s (1992) monthly data from January 1955 through May 1990. In doing so, we use three (non-parametric) inference methods: the BDS (Brock et al., 1996) test for whiteness (independent and identically distributed observations), the Lyapunov exponent estimator of Nychka et al. (1992), as well as the Lyapunov exponent estimator of Gencay and Dechert (1992). All of these tests are explicitly derived for use with noisy data.

The paper is organized as follows. Section 2 describes the data and provides statistical characteristics of East European black market exchange-rate changes. Section 3 provides a description of the key features of the three tests, focusing explicit attention on each test’s ability to detect chaos. Section 4 analyses the univariate properties of the seven East European black market exchange rate series and presents the results of the BDS test, the Nychka et al. (1992) and Gencay and Dechert (1992) Lyapunov exponent tests.
The final section concludes the paper with a discussion of the implications of the results.

2. The data

We study seven markets in this paper, those of the Polish zloty, Rumanian lei, Hungarian forint, Bulgarian lev, Czechoslovak koruna, Russian ruble, and East German mark. The time period of the analysis extends from January 1955 through May 1990. The exchange rates are the (end-of-month) black market exchange rates (expressed in U.S. dollars) reported in Koedijk and Kool (1992). The Bulgarian lev and the Russian ruble exchange rates have been adjusted for the currency reforms that took place in December 1961 and January 1961 in Bulgaria and the U.S.S.R., respectively—see Koedijk and Kool (1992).

Figure A provides graphical representations of the seven exchange rate series and Table 1 reports some summary statistics for exchange-rate changes. These changes appear to display the same behaviour as floating exchange-rate changes. In particular, the skewness numbers are consistent with symmetry but the kurtosis numbers point to significant deviations from normality for all countries—there are too many large changes to be consistent with normality. The column marked $S(0)$ provides estimates of the standardized spectral density function at the zero frequency based on the Bartlett window with the window size taken to be twice the square root of the number of observations. This gives consistent estimates of Cochrane’s (1988) measure of persistence, providing a useful diagnostic on the relative importance of permanent and transitory components. The point estimates suggest that with the exception of the Polish market all markets exhibit some degree of mean reversion.

3. Tests for chaos

Recently, five highly regarded tests for non-linearity or chaos (against various alternatives) have been introduced—see Barnett et al. (1995, 1997) for a detailed discussion. All five of the tests are purported to be useful with noisy data of moderate sample sizes. The tests are: the Hinich (1982) bispectrum test; the BDS [Brock et al. (1996)] test; White’s (1989) neural network test; Kaplan’s (1994) test; and the Nychka et al. (1992) dominant Lyapunov exponent estimator. Another very promising test [that is, similar in some respects to the Nychka et al. (1992) test] has also been recently proposed by Gencay and Dechert (1992). It is to be noted, however, that the Hinich bispectrum test, the BDS
FIGURE A. Logged East European black market exchange rates.
Table 1: Summary statistics for the first differences of the logarithms of East European black market exchange rates

<table>
<thead>
<tr>
<th>Series</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Skewness</th>
<th>Kurtosis-3</th>
<th>S(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polish złoty</td>
<td>0.012</td>
<td>0.099</td>
<td>-0.470</td>
<td>0.519</td>
<td>0.589</td>
<td>4.177</td>
<td>1.622 (0.589)</td>
</tr>
<tr>
<td>Romanian leu</td>
<td>0.003</td>
<td>0.075</td>
<td>-0.287</td>
<td>0.287</td>
<td>-0.003</td>
<td>1.733</td>
<td>0.422 (0.153)</td>
</tr>
<tr>
<td>Hungarian forint</td>
<td>0.0008</td>
<td>0.074</td>
<td>-0.356</td>
<td>0.566</td>
<td>1.248</td>
<td>11.286</td>
<td>0.459 (0.167)</td>
</tr>
<tr>
<td>Bulgarian leu</td>
<td>0.002</td>
<td>0.073</td>
<td>-0.247</td>
<td>0.352</td>
<td>0.778</td>
<td>3.644</td>
<td>0.547 (0.198)</td>
</tr>
<tr>
<td>Czechoslovak koruna</td>
<td>-0.0002</td>
<td>0.057</td>
<td>-0.189</td>
<td>0.263</td>
<td>0.528</td>
<td>2.076</td>
<td>0.648 (0.235)</td>
</tr>
<tr>
<td>Russian ruble</td>
<td>0.004</td>
<td>0.115</td>
<td>-0.811</td>
<td>1.386</td>
<td>2.769</td>
<td>55.346</td>
<td>0.322 (0.117)</td>
</tr>
<tr>
<td>East German mark</td>
<td>-0.002</td>
<td>0.075</td>
<td>-0.300</td>
<td>0.510</td>
<td>0.492</td>
<td>7.147</td>
<td>0.444 (0.161)</td>
</tr>
</tbody>
</table>

NOTE: Sample period, monthly data, 1955 M1–1990 M5. S(0) is a Bartlett estimate of the spectral density at zero frequency using a window size of $2\sqrt{N}$, where $N$ is the number of observations. Numbers in parentheses are standard errors.
test, White's test, and Kaplan's test are currently in use for testing non-linear dependence [whether chaotic (i.e. non-linear deterministic) or stochastic], which is necessary but not sufficient for chaos. Only the Nychka et al. (1992) and Gencay and Dechert (1992) tests are specifically focused on chaos as the null hypothesis.

In what follows, we apply only the BDS test and the Lyapunov exponent estimators of Nychka et al. (1992) and Gencay and Dechert (1992). The BDS test is currently the best available test for whiteness and can be used to test for residual non-linear dependence, after any linear structure has been removed through pre-whitening. The Nychka et al. (1992) Jacobian-based method involves the use of a neural network (or equivalently neural-net) method to estimate a map function by non-linear least squares, and subsequently the use of the estimated map and the data to produce an estimate of the dominant Lyapunov exponent. The Gencay and Dechert (1992) approach involves estimating all Lyapunov exponents of an unknown m-dimensional dynamical system. In what follows we describe these tests.

3.1. THE BDS TEST

The BDS test is based on the Grassberger and Procaccia (1983) correlation integral as the test statistic. In particular, under the null hypothesis of whiteness (independent and identically distributed observations), the BDS statistic is

\[ W(T, M, \varepsilon) = \frac{\sqrt{T} \left( C(T, m, \varepsilon) - C(T, 1, \varepsilon)^m \right)}{\sigma(T, m, \varepsilon)} \]

where \( C(T, m, \varepsilon) \) is the correlation function (integral), \( T = N - m + 1 \), \( N \) is the length of the series, \( m \) is the embedding dimension, \( \varepsilon \) is a sufficiently small number, and \( \sigma(T, m, \varepsilon) \) is an estimate of the asymptotic standard error of \( \{ C(T, m, \varepsilon) - C(T, 1, \varepsilon)^m \} \). The BDS statistic converges in distribution to a standardized normal random variable—see Brock et al. (1996) for details.

Since the asymptomatic distribution of the BDS test statistic is known under the null hypothesis of whiteness, the BDS test provides a direct (formal) statistical test for whiteness against general dependence, which includes both non-white linear and non-white non-linear dependence. Hence, the BDS test does not currently provide a direct test for non-linearity or for chaos, since the sampling distribution of the test statistic is not known (either in finite samples or asymptotically) under the null hypothesis of non-linearity, linearity, or chaos. It is, however, possible to use the BDS test to produce indirect evidence about non-linear dependence, which is necessary but not sufficient for chaos—see Barnett et al.
(1995, 1997), and Barnett and Hinich (1992) for a discussion of these issues.

3.2. THE NYCHKA et al. (1992) LYAPUNOV EXPONENT ESTIMATOR

The distinctive feature of chaotic systems is sensitive dependence on initial conditions (see, for example, Eckmann & Ruelle, 1985)—that is, exponential divergence of trajectories with similar initial conditions. The most important tool for diagnosing the presence of sensitive dependence on initial conditions (and thereby of chaoticity) in a dynamical system is provided by the dominant Lyapunov exponent, \( \lambda \). This exponent measures average exponential divergence or convergence between trajectories that differ only in having an "infinitesimally small" difference in their initial conditions and remains well-defined for noisy systems—see Kifer (1986). A bounded system with a positive Lyapunov exponent is one operational definition of chaotic behaviour.

We assume that the data \( \{x_t\} \) are real-valued and are generated by a non-linear autoregressive model of the form

\[
x_t = f(x_{t-L}, x_{t-2L}, \ldots, x_{t-mL}) + e_t
\]

(1)

where \( L \) is the time-delay parameter, \( m \) is the length of the autoregression, and \( e_t \) is a sequence of zero mean (and unknown constant variance) independent random variables. A state-space representation of (1) can be written as follows

\[
\begin{pmatrix}
  x_t \\
  x_{t-L} \\
  \vdots \\
  x_{t-mL+L}
\end{pmatrix}
= \begin{pmatrix}
  f(x_{t-L}, \ldots, x_{t-mL}) \\
  x_{t-L} \\
  \vdots \\
  x_{t-mL+L}
\end{pmatrix}
+ \begin{pmatrix}
  e_t \\
  0 \\
  \vdots \\
  0
\end{pmatrix}
\]

or equivalently,

\[
X_t + F(X_{t-L}) + E_t
\]

(2)

where

\[
X_t = (x_t, x_{t-L}, \ldots, x_{t-mL+L})^T, \quad F(X_{t-L}) = f((x_{t-L}, \ldots, x_{t-mL})^T), \quad x_{t-L}, \ldots, x_{t-mL+L})^T, \quad E_t = (e_t, 0, \ldots, 0)^T.
\]

The definition of the dominant Lyapunov exponent, \( \lambda \), can be formulated more precisely as follows. Let \( X_0, X_0' \in \mathbb{R}^m \) denote two "nearby" initial state vectors. After \( M \) iterations of model (2)
with the same random shock we have (using a truncated Taylor approximation)

$$\|X_M - X'_M\| = \|F^M(X_0) - F^M(X'_0)\| \approx \|(DF^M)_{X_0}(X_0 - X'_0)\|$$

where $F^M$ is the $M$th iterate of $F$ and $(DF^M)_{X_0}$ is the Jacobian matrix of $F$ evaluated at $X_0$. By application of the chain rule for differentiation, it is possible to show that

$$\|X_M - X'_M\| \approx \|T_M(X_0 - X'_0)\|$$

where $T_M = J_M J_{M-1} \ldots J_1$ and $J_1 = (DF)_{X_0}$. Letting $v_1(M)$ denote the largest eigenvalue of $T_M^T T_M$ the formal definition of the dominant Lyapunov exponent, $\lambda$, is

$$\lambda = \lim_{M \to \infty} \frac{1}{2M} \ln |v_1(M)|$$

In this setting, $\lambda$ gives the long-term rate of divergence or convergence between trajectories. A positive $\lambda$ measures exponential divergence of two nearby trajectories [and is often used as a definition of chaos—see, for example, Deneckere & Pelikan (1986)], whereas a negative $\lambda$ measures exponential convergence of two nearby trajectories.

In the next section we use the Nychka et al. (1992) Jacobian-based method and the LENNS program [see Ellner et al. (1992)] to estimate the dominant Lyapunov exponent. In particular we use a neural network model to estimate $f$ by non-linear least squares, and use the estimated map $f$ and the data $\{x_t\}$ to produce an estimate of the dominant Lyapunov exponent. In doing so, we follow the protocol described in Nychka et al. (1992).

The predominant model in statistical research on neural nets is the single (hidden) layer feedforward network with a single output. In the present context it can be written as

$$\hat{f}(X_t, \theta) = \alpha + \sum_{j=1}^{k} \beta_j \Psi(\omega_j + \gamma_j^T X_t)$$

where $X \in \mathbb{R}^m$ is the input, $\Psi$ is a known (hidden) univariate non-linear “activation function” [usually the logistic distribution function $\Psi(u) = 1/(1 + \exp(-u))$—see, for example, Nychka et al. (1992) and Gencay & Dechert (1992)], $\theta = (\alpha, \beta, \omega, \gamma)$ is the parameter vector, and $\gamma_j = (\gamma_{1j}, \gamma_{2j}, \ldots, \gamma_{mj})^T$. $\beta \in \mathbb{R}^k$ represents hidden unit weights and $\omega, \gamma \in \mathbb{R}^{k \times m}$ represent input weights to the hidden units. $k$ is the number of units in the hidden layer of the neural net. Notice that there are $[k(m + 2) + 1]$ free parameters in this model.
Given a data set of inputs and their associated outputs, the network parameter vector, $\theta$, is fitted by non-linear least squares to formulate accurate map estimates. As appropriate values of $L$, $m$, and $k$, are unknown, LENNS selects the value of the triple $(L,m,k)$ that minimizes the Bayesian Information Criterion (BIC)—see Schwartz (1978). Gallant and White (1992) have shown that we can then use, $J$, the estimate of the Jacobian matrix $J$, obtained from the approximate map $f$, as a non-parametric estimator of $J$. The estimate of the dominant Lyapunov exponent then is

$$\hat{\lambda} = \frac{1}{2N} \ln |\hat{v}_1(N)|$$

where $\hat{v}_1(N)$ is the largest eigenvalue of $T_N^2 T_N$ and where $T_N = J_N J_{N-1} \ldots J_1$.

### 3.3. The Gencay and Dechert (1992) Approach

Following Gencay and Dechert (1992), for the $m$-dimensional system (2), there are $m$-exponents ranked from largest to smallest

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_m.$$ 

Moreover, associated with each exponent, $j = 1, 2, \ldots, m$, there are nested subspaces $V_j \subset \mathbb{R}^m$ of dimension $m + 1 - j$ and with the property that

$$\lambda_j = \lim_{t \to \infty} \frac{1}{t} \ln \| (DF)^j_{x_0} v \|$$

for all $v \in V \setminus V^{j+1}$—see Guckenheimer and Holmes (1983: p. 256) for additional properties of Lyapunov exponents. Since

$$(DF)^j_{x_0} = (DF)_{x_0} (DF)_{x_0-1} \ldots (DF)_{x_0}$$

Gencay and Dechert (1992) calculate all the Lyapunov exponents by evaluating the Jacobian of the function $F$ along a trajectory, $\{\hat{v}_i\}$—see Gencay and Dechert (1992) for more details.

In the next section we use the NETLE program [see Kuan, Lin, & Gencay (1996)] to estimate all Lyapunov exponents. The estimation is carried out, as in Nychka et al. (1992), by a multivariate feedforward network estimation technique.

### 4. Empirical results

Before conducting non-linear dynamical analysis the data must be rendered stationary, delinearized (by replacing the stationary data
with residuals from an autoregression of the data) and transformed (if necessary) as described in Baumol and Benhabib (1989). We begin by testing for the presence of a unit root in the autoregressive representation of each individual time series. Dickey–Fuller (DF) and Augmented Dickey–Fuller (ADF) tests of the null hypothesis that a single unit root exists in the logarithm of each exchange rate series are conducted using the following ADF regression [see Dickey & Fuller (1981) for more details].

\[
\Delta \log z_t = \alpha_0 + \alpha_1 t + \alpha_2 \log z_{t-1} + \sum_{i=1}^{l} c_i \Delta \log z_{t-i} + \epsilon_t
\]  

(3)

where \( z_t \) is the series under consideration and \( l \) is selected to be large enough to ensure that \( \epsilon_t \) is white noise. The null hypothesis of a single unit root is rejected if \( \alpha_2 \) is negative and significantly different from zero.

In practice, the appropriate order of the autoregression in the ADF test is rarely known. One approach would be to use a model selection procedure based on some information criterion. However, Said and Dickey (1984) showed that the ADF test is valid asymptotically if the order of the autoregression is increased with sample size \( N \) at a controlled rate \( N^{1/3} \). For the sample used, this translates into an order of 8. It is to be noted that for an order of zero the ADF reduces to the simple DF test. Also, the distribution of the \( t \)-test for \( \alpha_2 \) in Equation (3) is not standard; rather it is that given by Fuller (1976).

Table 2 contains DF and ADF tests of the null hypothesis that a single unit root exists in the logarithm of each exchange-rate series. According to these tests the unit root null hypothesis cannot be rejected. Table 3 reports results for a second unit root—that is, for a unit root in the first (logged) differences of the series. Clearly, the null hypothesis of a second unit root is rejected. Hence, we conclude that these series are characterized as I(1), i.e. having a stochastic trend.

Since a stochastic trend has been confirmed for each of the series, the data are rendered stationary by taking first differences of logarithms. Also, since we are interested in non-linear dependence, we remove any linear dependence in the stationary data by fitting the best possible linear model. In particular, we prefilter the growth rates by the following autoregression

\[
\Delta \log z_t = b_0 + \sum_{j=1}^{q} b_j \Delta \log z_{t-j} + \epsilon_t, \quad |I_{t-1} \sim N(0,w_0)
\]  

(4)

using for each series the number of lags, \( q \), for which the Ljung–Box
Table 2 Tests for a unit root in the logarithms of the series:

$$\Delta \log z_t = a_0 + a_1 t + a_2 \log z_{t-1} + \sum_{i=1}^{l} c_i \Delta \log z_{t-i} + e_t$$

<table>
<thead>
<tr>
<th>Series</th>
<th>Without trend</th>
<th>With trend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DF</td>
<td>ADF</td>
</tr>
<tr>
<td>Polish zloty</td>
<td>2.175</td>
<td>2.495</td>
</tr>
<tr>
<td>Rumanian lei</td>
<td>-0.873</td>
<td>1.028</td>
</tr>
<tr>
<td>Hungarian forint</td>
<td>-2.489</td>
<td>-1.940</td>
</tr>
<tr>
<td>Bulgarian lev</td>
<td>-1.251</td>
<td>0.907</td>
</tr>
<tr>
<td>Czechoslovak koruna</td>
<td>-2.469</td>
<td>-2.048</td>
</tr>
<tr>
<td>Russian ruble</td>
<td>-2.691</td>
<td>-1.314</td>
</tr>
<tr>
<td>East German mark</td>
<td>-2.736</td>
<td>-2.651</td>
</tr>
</tbody>
</table>

NOTES: Results are reported for an ADF statistic of order 8. All the series are in logs. The 95% critical values for the DF and ADF test statistics are -2.868 and -2.868, respectively, for the “without trend” version of the test; and -3.422 and -3.422 for the “with trend” version of the test.

Table 3 Tests for a unit root in the first differences of the logarithms of the series

<table>
<thead>
<tr>
<th>Series</th>
<th>Without trend</th>
<th>With trend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DF</td>
<td>ADF</td>
</tr>
<tr>
<td>Polish zloty</td>
<td>-20.730</td>
<td>-6.291</td>
</tr>
<tr>
<td>Hungarian forint</td>
<td>-18.696</td>
<td>-7.312</td>
</tr>
<tr>
<td>Czechoslovak koruna</td>
<td>-20.426</td>
<td>-8.091</td>
</tr>
<tr>
<td>Russian ruble</td>
<td>-23.415</td>
<td>-8.121</td>
</tr>
<tr>
<td>East German mark</td>
<td>-19.301</td>
<td>-7.413</td>
</tr>
</tbody>
</table>

NOTES: Results are reported for an ADF statistic of order 8. The 95% critical values for the DF and ADF test statistics are -2.868 and -2.868, respectively, for the “without trend” version of the test; and -3.422 and -3.422 for the “with trend” version of the test.

(1978) Q(36) statistic is not significant at the 5% level. This identifies q to be 1 for the Polish zloty and Russian ruble, 11 for the Rumanian lei and Bulgarian lev, 4 for the Hungarian forint, 7
TABLE 4 Diagnostics of AR models under the Ljung-Box (1978) Q(36) Test statistics:

\[
    \Delta \log z_t = b_0 + \sum_{j=1}^{q} b_j \Delta \log z_{t-j} + \epsilon_t | I_{t-1} \sim N(0, \omega_0)
\]

<table>
<thead>
<tr>
<th>Series</th>
<th>AR Lag, q</th>
<th>Q-statistic</th>
<th>ARCH</th>
<th>J-B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polish zloty</td>
<td>1</td>
<td>0.232</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Romanian lei</td>
<td>11</td>
<td>0.062</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Hungarian forint</td>
<td>4</td>
<td>0.061</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Bulgarian lev</td>
<td>11</td>
<td>0.068</td>
<td>0.406</td>
<td>0.000</td>
</tr>
<tr>
<td>Czechoslovak koruna</td>
<td>7</td>
<td>0.342</td>
<td>0.150</td>
<td>0.000</td>
</tr>
<tr>
<td>Russian ruble</td>
<td>1</td>
<td>0.350</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>East German mark</td>
<td>5</td>
<td>0.052</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

NOTES: The Q-statistic is distributed as a \( \chi^2(36) \) on the null of no autocorrelation. ARCH is Engle’s (1982) Autoregressive Conditional Heteroskedasticity (ARCH) test distributed as a \( \chi^2(1) \) on the null of no ARCH. The Jarque–Bera test statistic is distributed as a \( \chi^2(2) \) under the null hypothesis of normality.

for the Czechoslovak koruna, and 5 for the East German mark—see Table 4.

Although the autocorrelation diagnostics in Table 4 indicate that the chosen AR models adequately remove linear dependence in the stationary data, the ARCH test suggests the presence of a time-varying variance. Since variance-non-linearity could be generated by either a (stochastic) ARCH process or a deterministic process, in what follows we model the conditional variance (or predictable volatility) using Bollerslev’s (1986) generalized autoregressive conditional heteroskedasticity (GARCH) model and Nelson’s (1991) exponential GARCH (EGARCH) model. One important feature of what we are doing, however, is to present the results of a diagnostic test for checking the adequacy of these models and choose among the estimated GARCH and EGARCH models.

The GARCH model is a generalization of the pure ARCH model, originally due to Engle (1982) and is useful in detecting non-linear patterns in variance while not destroying any signs of deterministic structural shifts in a model—see, for example, Lamoreux and Lastrapes (1990). Using the same AR structure as before we estimate the following GARCH (1,1) model
\[
\Delta \log z_t = b_0 + \sum_{j=1}^{q} b_j \Delta \log z_{t-j} + \epsilon_t, \quad I_{t-1} \sim N(0, \sigma_t^2)
\]  
\[
\sigma_t^2 = w_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2
\]

where \(N(0, \sigma_t^2)\) represents the normal distribution with mean zero and variance \(\sigma_t^2\). Parameter estimates and diagnostic tests are given in Table 5. First, estimated coefficients of the ARCH term, \(\alpha_1\), and the GARCH term, \(\beta_1\), are positive and significant at the 5% level. Also, the \(Q\)-test finds no linear dependence and the ARCH test finds no ARCH effects, suggesting that the lag structure of the conditional variance is correctly identified. Moreover, the null hypothesis that \(\alpha_1 + \beta_1 = 1\) is rejected, suggesting the absence of integrated variances.

GARCH models assume that the conditional variance in Equation (5) is a function only of the magnitude of the lagged residuals and not their signs—i.e. only the size, not the sign, of lagged residuals determines conditional variance. This assumption imposes important limitations on GARCH models. For example, these models are not well suited to capture the so-called “leverage effect”, first noted by Black (1976). To meet these objections, we use Nelson’s (1991) exponential GARCH (1,1), or EGARCH (1,1), also inspired by Engle’s (1982) ARCH model, in which the conditional variance \(\sigma_t^2\) depends on both the size and the sign of lagged residuals as follows

\[
\log \sigma_t^2 = w_0 + \beta \log(\sigma_{t-1}^2) + \alpha \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}}.
\]

The log transformation ensures that \(\sigma_t^2\) remains non-negative for all \(t\). Clearly, the impact of the most recent residual is now exponential rather than quadratic.

Parameter estimates and diagnostic tests for the EGARCH (1, 1) model are presented in Table 6. In general, \(\beta\) is statistically less than unity except for the East German mark and (to a lesser extent) the Russian ruble, implying that variances are not integrated and that unconditional variances are finite. Moreover, except for the Russian ruble and the East German mark, the log likelihood for the EGARCH (1,1) model is higher than that for the GARCH (1, 1) model, suggesting that the EGARCH model is superior to the GARCH model for these series. To investigate this further, and in order to choose between GARCH and EGARCH models, we present in Table 7 the results of a diagnostic test suggested by Kearns and Pagan (1993) for checking the adequacy of these models. The test involves the regression of \(\epsilon_t^2\) against a constant and the estimated
Table 5 GARCH (1,1) parameter estimates and error term diagnostics:

\[
\Delta \log z_t = b_0 + \sum_{j=1}^\infty b_j \Delta \log z_{t-j} + e_t \mid I_{t-1} \sim N(0, \sigma_t^2), \sigma_t^2 = w_0 + a_1 e_{t-1}^2 + b_1 \sigma_{t-1}^2
\]

<table>
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<tr>
<th>Series</th>
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<th>(w_0)</th>
<th>(a_1)</th>
<th>(b_1)</th>
<th>(Q)-statistic</th>
<th>(Q(e^2))</th>
<th>ARCH</th>
<th>J-B</th>
<th>Log L</th>
<th>(z_1 + b_1 = 1)</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>0.001 (2.3)</td>
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<td>0.741 (9.9)</td>
<td>0.278</td>
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<td>0.988</td>
<td>0.000</td>
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<td>0.000</td>
</tr>
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<td>0.162 (2.6)</td>
<td>0.751 (9.6)</td>
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<td>0.000</td>
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</tr>
<tr>
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<td>11</td>
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<td>0.488 (3.3)</td>
<td>-0.096 (3.2)</td>
<td>0.000</td>
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<td>0.000 (2.5)</td>
<td>0.413 (2.3)</td>
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<td>0.000</td>
<td>613.982</td>
<td>0.000</td>
</tr>
<tr>
<td>Russian rouble</td>
<td>1</td>
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<td>0.004 (1.4)</td>
<td>0.477 (2.7)</td>
<td>0.793</td>
<td>0.024</td>
<td>0.028</td>
<td>0.000</td>
<td>479.776</td>
<td>0.001</td>
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<td>East German mark</td>
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<td>0.241 (1.9)</td>
<td>0.015 (0.1)</td>
<td>0.063</td>
<td>0.945</td>
<td>0.976</td>
<td>0.000</td>
<td>520.144</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: Numbers in parentheses next to the GARCH (1,1) parameter estimates are absolute t-ratios. The Q-statistic is distributed as a \(\chi^2(36)\) on the null of no autocorrelation. The ARCH statistic is distributed as a \(\chi^2(1)\) on the null of no ARCH. The Jarque–Bera test statistic is distributed as a \(\chi^2(2)\) under the null hypothesis of normality.
Table 6. **EGARCH (1,1) parameter estimates and error term diagnostics:**

\[
\Delta \log z_t = b_0 + \sum_{j=1}^{q} b_j \Delta \log z_{t-j} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_t^2), \quad \log \sigma_t^2 = w_0 + \beta \log(\sigma_{t-1}^2) + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \frac{\epsilon_{t-1}^2}{\sigma_{t-1}}.
\]

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<th>AT Lag</th>
<th>( w_0 )</th>
<th>( \alpha )</th>
<th>( \gamma )</th>
<th>( \beta )</th>
<th>( Q )-statistic</th>
<th>( Q(q^2) )</th>
<th>ARCH</th>
<th>( J-B )</th>
<th>Log L</th>
<th>( \beta=1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polish zloty</td>
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<td>0.196 (3.6)</td>
<td>0.230 (3.9)</td>
<td>0.897 (24.9)</td>
<td>0.221</td>
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<td>0.701</td>
<td>0.000</td>
<td>428.007</td>
<td>0.004</td>
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<tr>
<td>Romanian lei</td>
<td>11</td>
<td>-1.001 (2.7)</td>
<td>0.362 (3.1)</td>
<td>0.002 (0.0)</td>
<td>0.864 (9.6)</td>
<td>0.060</td>
<td>0.951</td>
<td>0.236</td>
<td>0.000</td>
<td>517.297</td>
<td>0.021</td>
</tr>
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<td>0.064 (9.6)</td>
<td>0.702 (2.8)</td>
<td>-0.056 (0.5)</td>
<td>0.587 (4.7)</td>
<td>0.008</td>
<td>0.254</td>
<td>0.053</td>
<td>0.000</td>
<td>550.115</td>
<td>0.001</td>
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<tr>
<td>Bulgarian lev</td>
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<td>-0.053 (3.1)</td>
<td>-0.085 (1.6)</td>
<td>0.063 (2.4)</td>
<td>0.997 (110.0)</td>
<td>0.111</td>
<td>0.995</td>
<td>0.328</td>
<td>0.000</td>
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<td>0.017</td>
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<td>Czechoslovak koruna</td>
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<td>0.460 (4.3)</td>
<td>0.056 (0.7)</td>
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<td>0.453</td>
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<td>0.010</td>
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<td>Russian ruble</td>
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<td>-1.817 (1.9)</td>
<td>0.488 (1.8)</td>
<td>-0.135 (0.7)</td>
<td>0.706 (4.1)</td>
<td>0.790</td>
<td>0.046</td>
<td>0.065</td>
<td>0.000</td>
<td>473.765</td>
<td>0.085</td>
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<tr>
<td>East German mark</td>
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<td>-3.610 (1.5)</td>
<td>0.386 (2.4)</td>
<td>-0.006 (0.1)</td>
<td>0.367 (0.8)</td>
<td>0.075</td>
<td>0.983</td>
<td>0.904</td>
<td>0.000</td>
<td>516.141</td>
<td>0.177</td>
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</tbody>
</table>

NOTES: Numbers in parentheses next to the EGARCH (1,1) parameter estimates are absolute \( t \)-ratios. The \( Q \)-statistic is distributed as a \( \chi^2(36) \) on the null of no autocorrelation. The ARCH statistic is distributed as a \( \chi^2(1) \) on the null of no ARCH. The Jarque–Bera test statistic is distributed as a \( \chi^2(2) \) under the null hypothesis of normality.
Table 7. Comparison of predictive power for the conditional variance of exchange rate changes: \( \tilde{\sigma}_t^2 = b_0 + b_1 \tilde{\sigma}_{t-1}^2 + \zeta_t \)

<table>
<thead>
<tr>
<th>Series</th>
<th>GARCH (1,1) results</th>
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<th></th>
<th>EGARCH (1,1) results</th>
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<tbody>
<tr>
<td></td>
<td>( b_0 )</td>
<td>( b_1 )</td>
<td>( R^2 )</td>
<td>( Q )-statistic</td>
<td>( b_0 )</td>
<td>( b_1 )</td>
</tr>
<tr>
<td>Polish zloty</td>
<td>0.003 (2.4)</td>
<td>0.625 (3.3)</td>
<td>0.069</td>
<td>0.248</td>
<td>-0.001 (0.7)</td>
<td>1.135 (1.1)</td>
</tr>
<tr>
<td>Rumanian lei</td>
<td>0.001 (1.4)</td>
<td>0.731 (1.9)</td>
<td>0.061</td>
<td>0.325</td>
<td>0.000 (0.5)</td>
<td>0.908 (0.5)</td>
</tr>
<tr>
<td>Hungarian forint</td>
<td>-0.000 (0.5)</td>
<td>1.100 (1.0)</td>
<td>0.283</td>
<td>0.042</td>
<td>-0.005 (4.9)</td>
<td>2.179 (7.3)</td>
</tr>
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<td>Bulgarian lev</td>
<td>-0.001 (0.5)</td>
<td>1.249 (0.6)</td>
<td>0.026</td>
<td>0.967</td>
<td>-0.001 (0.8)</td>
<td>1.282 (1.1)</td>
</tr>
<tr>
<td>Czechoslovak koruna</td>
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<td>0.279 (5.6)</td>
<td>0.012</td>
<td>0.504</td>
<td>0.002 (3.1)</td>
<td>0.389 (3.9)</td>
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<tr>
<td>Russian ruble</td>
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<td>0.999</td>
<td>-0.031 (7.6)</td>
<td>5.357 (18.7)</td>
</tr>
<tr>
<td>East German mark</td>
<td>-0.001 (0.9)</td>
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<td>0.000</td>
<td>0.444</td>
<td>0.001 (0.5)</td>
<td>0.899 (0.5)</td>
</tr>
</tbody>
</table>

NOTES: Absolute t-statistics for \( b_0 = 0 \) and \( b_1 = 1 \) are in parentheses. \( R^2 \) is the coefficient of determination. \( Q(36) \) is the Ljung–Box statistic for 36 lags of the residual autocorrelation.
conditional variance $\sigma_r^2$. The intercept of such a regression should be zero and the slope coefficient unity.

The insignificant $Q(36)$ statistic in Table 7 indicates that each of these models captures much of the persistence in actual volatility and the coefficient of determination indicates how well the estimated conditional variance predicts the actual variance and is used to compare the GARCH and EGARCH models. On the basis of these results, a comparison between the log likelihood values in Tables 6 and 7, and the fact that the EGARCH variances for the East German mark and Russian ruble are integrated, in what follows we test for whiteness and chaos using the standardized EGARCH residuals for each of the Polish zloty, the Rumanian lei, the Hungarian forint, the Bulgarian lev, and the Czechoslovak koruna and the standardized GARCH residuals for the Russian ruble and the East German mark—the standardized residuals are defined as $\varepsilon_t/\sigma_t$, where $\varepsilon_t$ is the residual of the mean equation and $\sigma_t^2$ its estimated (time-varying) variance.

We now apply the BDS test of whiteness to the raw (log differenced) data, the AR residuals, and the standardized residuals. In doing so, we use the BDS to test the null hypothesis of independent random variables (against the alternative of non-independent random variables, thereby treating the BDS as an omnibus two-sided test). The results are presented in Table 8 for dimensions 2 through 5 and $\varepsilon$ equalling 0.5, 1.5, and 2 standard deviations of the data. In the case of the raw data and AR residuals we use the asymptotic distribution of the BDS whereas in the case of the standardized residuals we use Hsieh’s (1991, Table XIII) simulated BDS critical values for EGARCH standardized residuals. In this regard, Brock (1986) proved that the asymptotic distribution of the BDS test statistics is not altered by using residuals instead of raw data in linear models and his theorem can also be extended to residuals of some non-linear models [such as, for example, the non-linear moving average model, proposed by Robinson (1977), but not the ARCH-type models].

In every case the BDS test on the raw data rejects the null hypothesis of independent and identically distributed observations, a finding which is consistent with deterministic chaos. Moreover, the BDS test statistics for the AR residuals do not differ substantially from those using the raw data, suggesting that the BDS test is not merely picking up linear dependence, but is in fact detecting strong non-linear dependence in the data. The ARCH-type models, however, remove considerable serial dependence from the raw data. In particular, the BDS statistics for the standardized residuals rejects the null of independent and identically distributed observations for only the Bulgarian lev, the Czechoslovak koruna, and the East German mark, which is consistent with deterministic chaos.
Table 8. BDS statistics, at dimensions 2 through 5 and \( \varepsilon \) equalling 0.5, 1, 1.5 and 2 standard deviations of the data

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<th>( m )</th>
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<td>data residuals residuals data residuals residuals data residuals residuals data residuals residuals</td>
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<thead>
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<th></th>
<th>Russian ruble</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>5.597*</td>
<td>7.415*</td>
<td>1.070*</td>
<td>6.151*</td>
<td>6.759*</td>
<td>0.7621</td>
<td>8.182*</td>
<td>7.607*</td>
<td>0.8030</td>
<td>11.127*</td>
<td>9.807*</td>
<td>1.4434</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6.175*</td>
<td>7.783*</td>
<td>0.2441</td>
<td>6.810*</td>
<td>7.568*</td>
<td>0.2118</td>
<td>8.200*</td>
<td>8.275*</td>
<td>0.1276</td>
<td>10.451*</td>
<td>10.241*</td>
<td>0.4957</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>6.535*</td>
<td>8.106*</td>
<td>0.0303</td>
<td>7.147*</td>
<td>7.384*</td>
<td>0.1485</td>
<td>8.129*</td>
<td>8.234*</td>
<td>0.2004</td>
<td>9.827*</td>
<td>9.914*</td>
<td>0.1971</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.736*</td>
<td>8.303*</td>
<td>0.2106</td>
<td>7.312*</td>
<td>8.060*</td>
<td>0.1413</td>
<td>8.112*</td>
<td>8.364*</td>
<td>0.4963</td>
<td>9.340*</td>
<td>9.672*</td>
<td>0.3957</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>East German mark</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

**NOTE:** An asterisk indicates significance at the 5% (two-tailed) level. In the case of the raw data and AR residuals we use the asymptotic distribution of the BDS. In the case of the GARCH standardized residuals we use Hsieh’s (1991, Table XIII) simulated BDS critical values for EGARCH standardized residuals.
With the Nychka et al. (1992) Lyapunov exponent test, the Bayesian Information Criterion (BIC) point estimates of the dominant Lyapunov exponent for each parameter triple \((L, m, k)\) are displayed in Table 9 along with the respective optimized value of the BIC criterion. Clearly, all but two Lyapunov exponent point estimates are negative, supporting the conclusion that only the Russian ruble and the East German mark have a chaotic nonlinear generating process.

Of course, the standard errors of the estimated dominant Lyapunov exponents are not known [there has not yet been any published research on the computation of a standard error for the Nychka et al. (1992) Lyapunov exponent estimate]. It is possible, however, to produce sensitivity plots that are informative about precision, as the ones in Figure B. Figure B indicates the sensitivity of the dominant Lyapunov exponent estimate to variations in the parameters, by plotting the estimated dominant Lyapunov exponent for each setting of \((L, m, k)\), where \(L = 1, 2, 3, m = 1, \ldots, 10,\) and \(k = 1, 2, 3\).

Finally, in Table 10 we present Lyapunov exponent estimates for the standardized residuals using the algorithm proposed by Gencay and Dechert (1992), for embedding dimensions, \(m = 1, 2, 3, 4\). The Schwartz-information criterion (SIC) is used to determine the optimal complexity of the feedforward network. In fact, for each embedding dimension, four network complexities are estimated by allowing the hidden unit to change from \(k = 2, 3, 4,\)

### Table 9. The Nychka et al. (1992) BIC section of the parameter triple \((L, m, k)\), the value of the minimized BIC, and the dominant Lyapunov exponent point estimate

<table>
<thead>
<tr>
<th>Series</th>
<th>((L, m, k))</th>
<th>Value of the minimized BIC</th>
<th>Dominant Lyapunov exponent point estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polish zloty</td>
<td>(3,1,1)</td>
<td>1.4425</td>
<td>-4.247</td>
</tr>
<tr>
<td>Rumanian lei</td>
<td>(2,1,1)</td>
<td>1.4440</td>
<td>-4.356</td>
</tr>
<tr>
<td>Hungarian forint</td>
<td>(2,1,1)</td>
<td>1.4465</td>
<td>-4.365</td>
</tr>
<tr>
<td>Bulgarian lev</td>
<td>(2,1,1)</td>
<td>1.4409</td>
<td>-5.216</td>
</tr>
<tr>
<td>Czechoslovak koruna</td>
<td>(1,8,2)</td>
<td>1.4382</td>
<td>-0.011</td>
</tr>
<tr>
<td>Russian ruble</td>
<td>(1,10,2)</td>
<td>1.4066</td>
<td>0.075</td>
</tr>
<tr>
<td>East German mark</td>
<td>(3,6,2)</td>
<td>1.3764</td>
<td>0.041</td>
</tr>
</tbody>
</table>

**NOTE:** Numbers in parentheses represent the BIC selection of the parameter triple, \((L, m, k)\), where \(L\) is the time delay parameter, \(m\) is the number of lags in the autoregression and \(k\) is the number of units in the hidden layer of the neural net.
and 5. In all cases, $k=2$ provides the lowest SIC value although the inferences implied by all network complexities are robust across all values of $k$. The results indicate that for all series,
Table 10. Lyapunov exponent estimates based on the Genca and Dechert (1992) algorithm

<table>
<thead>
<tr>
<th>Hiddens, k</th>
<th>SIC 1</th>
<th>λ₁</th>
<th>SIC 1</th>
<th>λ₁</th>
<th>λ₂</th>
<th>SIC 1</th>
<th>λ₁</th>
<th>λ₂</th>
<th>λ₃</th>
<th>SIC 1</th>
<th>λ₁</th>
<th>λ₂</th>
<th>λ₃</th>
<th>λ₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.079</td>
<td>-1.380</td>
<td>0.104</td>
<td>-4.567</td>
<td>-7.200</td>
<td>0.112</td>
<td>-0.902</td>
<td>-1.064</td>
<td>-1.521</td>
<td>0.175</td>
<td>-1.082</td>
<td>-1.548</td>
<td>-2.124</td>
<td>-2.696</td>
</tr>
<tr>
<td>3</td>
<td>0.120</td>
<td>-1.384</td>
<td>0.163</td>
<td>-6.547</td>
<td>-9.640</td>
<td>0.194</td>
<td>-1.131</td>
<td>-1.703</td>
<td>-2.602</td>
<td>0.210</td>
<td>-0.185</td>
<td>-0.266</td>
<td>-0.358</td>
<td>-0.523</td>
</tr>
<tr>
<td>4</td>
<td>0.162</td>
<td>-6.333</td>
<td>0.201</td>
<td>-1.502</td>
<td>-3.097</td>
<td>0.229</td>
<td>-0.525</td>
<td>-1.110</td>
<td>-1.726</td>
<td>0.307</td>
<td>-0.450</td>
<td>-0.602</td>
<td>-0.789</td>
<td>-1.100</td>
</tr>
<tr>
<td>5</td>
<td>0.192</td>
<td>-1.440</td>
<td>0.217</td>
<td>-0.347</td>
<td>-0.935</td>
<td>0.297</td>
<td>-0.629</td>
<td>-1.008</td>
<td>-1.153</td>
<td>0.375</td>
<td>-0.229</td>
<td>-0.303</td>
<td>-0.480</td>
<td>-0.740</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>m = 2</th>
<th>m = 3</th>
<th>m = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polish zloty</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.083</td>
<td>-2.329</td>
<td>0.108</td>
</tr>
<tr>
<td>3</td>
<td>0.114</td>
<td>-3.038</td>
<td>0.167</td>
</tr>
<tr>
<td>4</td>
<td>0.175</td>
<td>-3.202</td>
<td>0.202</td>
</tr>
<tr>
<td>5</td>
<td>0.201</td>
<td>-2.902</td>
<td>0.255</td>
</tr>
</tbody>
</table>

| Rumanian lei |       |       |       |
| 2         | 0.061 | -14.932 | 0.121 | -1.455 | -1.585 | 0.155 | -1.132 | -1.563 | -1.727 | 0.183 | -2.014 | -2.900 | -4.109 | -4.745 |
| 3         | 0.100 | -8.585 | 0.150 | -0.929 | -1.830 | 0.178 | -0.697 | -1.020 | -1.834 | 0.177 | -0.175 | -0.515 | -0.798 | -1.286 |
| 4         | 0.136 | -1.338 | 0.171 | -0.536 | -1.000 | 0.230 | -0.957 | -1.175 | -1.669 | 0.314 | -0.337 | -0.453 | -0.637 | -1.878 |
| 5         | 0.162 | -4.616 | 0.267 | -1.451 | -1.964 | 0.304 | -0.292 | -0.698 | -1.308 | 0.338 | -0.060 | -0.196 | -0.298 | -0.522 |

| Hungarian forint |       |       |       |
| 2         | 0.095 | -2.619 | 0.112 | -6.473 | -8.452 | 0.148 | -0.383 | -0.456 | -0.568 | 0.193 | -1.810 | -3.090 | -4.312 | -4.756 |
| 3         | 0.123 | -1.472 | 0.167 | -1.053 | -2.035 | 0.194 | -1.252 | -2.279 | -3.141 | 0.203 | -0.277 | -0.734 | -0.987 | -1.092 |
| 4         | 0.178 | -1.224 | 0.197 | -0.646 | -1.489 | 0.222 | -0.401 | -0.751 | -0.787 | 0.267 | -0.285 | -0.543 | -0.724 | -1.044 |
| 5         | 0.190 | -1.590 | 0.291 | -0.823 | -1.162 | 0.257 | -0.272 | -0.470 | -0.880 | 0.406 | -0.359 | -0.531 | -0.732 | -1.122 |

<p>| Bulgarian lev |       |       |       |
| 2         | 0.095 | -2.019 | 0.112 | -6.473 | -8.452 | 0.148 | -0.383 | -0.456 | -0.568 | 0.193 | -1.810 | -3.090 | -4.312 | -4.756 |
| 3         | 0.123 | -1.472 | 0.167 | -1.053 | -2.035 | 0.194 | -1.252 | -2.279 | -3.141 | 0.203 | -0.277 | -0.734 | -0.987 | -1.092 |
| 4         | 0.178 | -1.224 | 0.197 | -0.646 | -1.489 | 0.222 | -0.401 | -0.751 | -0.787 | 0.267 | -0.285 | -0.543 | -0.724 | -1.044 |
| 5         | 0.190 | -1.590 | 0.291 | -0.823 | -1.162 | 0.257 | -0.272 | -0.470 | -0.880 | 0.406 | -0.359 | -0.531 | -0.732 | -1.122 |</p>
<table>
<thead>
<tr>
<th>m</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Czechoslovak koruna</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.087</td>
<td>-3.281</td>
<td>0.122</td>
<td>-3.319</td>
</tr>
<tr>
<td>3</td>
<td>0.128</td>
<td>-1.511</td>
<td>0.134</td>
<td>-1.791</td>
</tr>
<tr>
<td>4</td>
<td>0.170</td>
<td>-2.795</td>
<td>0.173</td>
<td>-0.540</td>
</tr>
<tr>
<td>5</td>
<td>0.182</td>
<td>-3.683</td>
<td>0.240</td>
<td>-0.743</td>
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<tr>
<td>Russian ruble</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.098</td>
<td>-2.834</td>
<td>0.125</td>
<td>-9.162</td>
</tr>
<tr>
<td>3</td>
<td>0.134</td>
<td>-3.869</td>
<td>0.144</td>
<td>-0.502</td>
</tr>
<tr>
<td>4</td>
<td>0.173</td>
<td>-3.078</td>
<td>0.159</td>
<td>-0.604</td>
</tr>
<tr>
<td>5</td>
<td>0.147</td>
<td>-2.288</td>
<td>0.221</td>
<td>-0.512</td>
</tr>
<tr>
<td>East German mark</td>
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<td></td>
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<tr>
<td>2</td>
<td>0.088</td>
<td>-4.546</td>
<td>0.126</td>
<td>-7.189</td>
</tr>
<tr>
<td>3</td>
<td>0.131</td>
<td>-3.522</td>
<td>0.172</td>
<td>-2.324</td>
</tr>
<tr>
<td>4</td>
<td>0.170</td>
<td>-4.864</td>
<td>0.215</td>
<td>-0.847</td>
</tr>
<tr>
<td>5</td>
<td>0.185</td>
<td>-3.080</td>
<td>0.250</td>
<td>-1.033</td>
</tr>
</tbody>
</table>

**NOTES:** m refers to the embedding dimension. SIC is the Schwartz (1978) information criterion. For each value of m and for each number of hidden units, the Lyapunov exponents are ordered from largest to smallest.
all Lyapunov exponents are less than zero for all embedding dimensions.

Clearly, there is some consistency in inference across methods. In particular, the BDS test rejects the null hypothesis of independent and identically distributed observations for three of the seven markets—Bulgaria, Czechoslovakia, and East Germany—whereas the Lyapunov exponent estimator proposed by Nychka et al. (1992) supports the hypothesis of chaotic dynamics in two markets—East Germany and Russia. We also find some consistency between the Nychka et al. (1992) and the Gencay and Dechert (1992) Lyapunov exponent estimators—the two tests that maintain chaos as the null hypothesis—although with the latter we find that all Lyapunov exponents are negative, perhaps suggesting a possible explanation for the controversies that exist regarding empirical evidence of chaos in economic and financial data.

5. Conclusion

This paper provides results of non-linear dynamical analysis of seven East European black market exchange rate series, using two of the most reputable (non-parametric) inference methods. We find only limited robustness across inference methods, which is consistent with the evidence reported in Barnett et al. (1995). The BDS test rejects the null hypothesis of independent and identically distributed observations for all the original data and all the AR residuals, while the standardized residuals appear not to be independent and identically distributed for only the Bulgarian lev, the Czechoslovak koruna, and the East German mark. With the Nychka et al. (1992) Lyapunov exponent test, however, we find evidence that the Russian ruble and East German mark exchange rates are chaotic. The results with the Gencay and Dechert (1992) estimator suggest that all Lyapunov exponents are negative for all series.

These findings have interesting implications. Most importantly, since chaos generates output that mimics the output of stochastic systems, the possible existence of chaos in the East German mark and Russian ruble exchange rate series could be exploitable and even invaluable. In particular, the existence of chaos implies that profitable, non-linearity-based trading rules may exist (at least in the short run and provided the actual generating mechanism is known exactly), perhaps raising questions about the efficient markets hypothesis. Prediction, however, over long periods is all but impossible, due to the sensitive dependence on initial conditions property of chaos. Of course, estimation of the unknown parameters of a chaotic map is currently extremely difficult, thereby reducing the potential for the use of chaotic dynamics in financial markets.
Acknowledgements

We would like to thank Ramo Gencay and Tung Liu for many helpful suggestions, three referees for comments that improved the paper, and Paul Dormaar for research assistance.

References


Ellner, S., Nychka, D.W. & Gallant, A.R. (1992). *LENNs, a program to estimate the dominant Lyapunov exponent of noisy nonlinear systems from time series*
data. Institute of Statistics Mimeo Series #2235 (BMA Series #39), Statistics Department, North Carolina State University, Raleigh, NC 27695–8203.


